Discussion Paper

Teaching Undergraduate Mathematics using CAS Technology: Issues and Prospects

By Patrick C. Tobin and Vida Weiss

Australian Catholic University, Melbourne Campus, Victoria, Australia
Patrick.Tobin@acu.edu.au

vweiss@swin.edu.au

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The use of handheld CAS technology in undergraduate mathematics courses in Australia is paradoxically shrinking under sustained disapproval or disdain from the professional mathematics community. Mathematics education specialists argue with their mathematics colleagues over a range of issues in course development and this use of CAS or even graphics calculator technology in assessment is a serious sticking point. We review some of the issues in the literature and established local practice and prospects for change within tertiary mathematics with reference to international concerns and the experience in the secondary sector. Problems identified could argue for increased technology use in service courses.

1 INTRODUCTION

Computer Algebra Systems (CAS) has generated extraordinary controversy in the past 25 years of their existence. Risser (2011) discussed the strong antagonism by professional mathematicians to the use of CAS technology and particularly portable calculator CAS versions in the teaching of undergraduate mathematics. Meagher (2012) also commented on the negative attitude to CAS technology use in class over a long period, noting reports as far back as 1999. This response from the professional mathematics communities in the US is mirrored in the Australian experience. There has been continued disapproval of any graphics calculator use in assessment from the Australian Mathematical Sciences Institute (AMSI) and these calculators have been generally disallowed from the start in all traditional mathematics courses. In-service mathematics courses the history of use in class and assessment has been more favourable but even there it was not widespread (Tobin and Weiss, 2011). In 2012 two universities, Swinburne University of Technology in Victoria and Edith Cowan University in Western Australia, which were previously entrenched users of the technology, decided to remove it from use in examination assessment situations in engineering mathematics ostensibly because of objections from some engineering and science lecturers. This practice is in contrast to policy commonly followed in the secondary school system where a natural incorporation of the technology has progressed in most states in Australia – a situation echoing that described by Madison (2001) and Artigue (2002).

2 RATIONALE FOR INVESTIGATING CHANGE

The divide over didactic issues between mathematics practitioners and the mathematics education community in Australia is fairly strong (Thomas, 2011). Unfortunately there has been insufficient dialogue on technology issues. It is reasonable for mathematics practitioners to express concern that technology is taken up without clear idea of consequences to conceptual development but it is also regrettable that prospects for improving or changing mathematics courses in line with real options are not always addressed. Risser (2011) suggests that concern over curriculum change is one obstacle to CAS use – but this is hardly a valid objection. Scientific calculators reduced our teaching of log tables as multiplication and division could be done directly which is more natural. Examining past courses shows that mathematics curricula have often changed as more relevant components are developed or accentuated.

Admittedly, there is conflicting evidence on trials of use (see eg. Tall, Smith and Piez, 2010 who outline research which is either favourable or neutral). There is no clear pattern for supporters of technology use as yet and so the conservative doctrine of minimal change has been firmly applied. In this case it is useful to explore reasons to reach for change.

One motive for using CAS is that it may offer a way to rejuvenate student interest. Mathematics courses have suffered substantial decline in popularity in the school and university system over the past four decades (ICE, 2006; National Strategic Review, 2006). This is a fact in the UK and Australia at any rate, is common worldwide and has trended for some time (see e.g. Kennedy, Lyons and Quinn, 2014; Varghese, 2008; Jackson, 2000). Reasons for this are varied. Mathematics is certainly a classically 'hard' subject and the well-known problems that students have at school level are exacerbated by both its poor image and frequently bad teaching – especially at primary and junior secondary levels. Reversing this will surely require universal specialist mathematics teaching at primary level. Removing maths teaching from the general classroom teacher would elevate standards of teaching practice and reinforce the status of the subject as both special and important. As a sequential subject, progress depends on mastery of prior concepts and failure to reach adequate level forever excludes a student from a large number of career pathways. The issue of poor teaching was addressed in the National Strategic Review (2006) where the problems at primary and junior secondary levels were highlighted and it was noted that even senior secondary student teachers come from a cohort with reduced major studies – about one quarter do not do classes above second year undergraduate level. This issue was revisited by Rubinstein (2009) when making a plea for a rethink on math education issues and programs at tertiary level. The attempt to revitalise mathematics teaching was basic to the Calculus Reform movement in the US which aimed to strengthen understanding through multiple representation of ideas and graphics calculators were a key tool in their process. Even use of graphics calculators with no CAS is discouraged or forbidden in test assessment in most Australian universities now.

Addressing poor teaching practice will not alone tackle the issue of poor image, although doubtless bad experiences in the subject and poor understanding make it much more unpopular with many students. The subject has also suffered in 'sexiness' compared with newer secondary subjects like psychology or the plethora of subjects ending in the word 'studies'. These subjects may be seen by us as 'soft' options but in the market place they are gaining strength over mathematics and science. In fact science in general was once seen as 'sexy' in that heady period of 20 years after the war when atomic energy, space exploration and the cracking of the genetic code made scientists respectable and important in public perception. Since that time, science has come under repeated, largely unfair, criticism for the problems besetting the world from environmental degradation to weapons of mass destruction. It seems the image of Frankenstein is replacing that of Einstein. With the competitive job market and seemingly more materialist world view, the student focus now is on chasing well paid careers and academia and science are not seen in this light (Kennedy et al, 2014). This is just the situation described in Germany by Jackson (2000). Most mathematics students who achieve top marks in year 12 mathematics head to courses leading to careers in medicine, law and business. Some will study engineering fortunately but fewer enrol directly into science courses. This does not matter providing we have enough in science and engineering although it would be desirable to have the best possible group – and there is evidence this is not the case in reducing entry scores and difficulty in attracting strong students.

The decline in student participation in year 12 mathematics courses was highlighted in a report issued in September 2006 (Barrington, 2006). Overall numbers of students in the top mathematics courses have declined from 14.1% to 11.7 % in the decade to 2004 and a similar decline was observed in the proportion of students in intermediate level courses – a drop from 27.2% to 22.6%. In the decade since then these figures have declined further to 9.4% and 19.4% respectively. (AMSI, 2014).

The senior secondary students of today represent a very different cohort from that present forty years ago. In that time the proportion of students in year 7 completing year 12 has increased substantially yet the proportion in high level mathematics courses has continually declined as noted. This issue has received general publicity within Australia. For example, the Adelaide Advertiser in August 2010 (Simos, 2010) reported a survey by the Technology Industry Association on 178 teachers (mainly from the public sector) regarding issues in school mathematics in South Australia. This survey comments on negative attitudes by students to mathematics, on inadequate training of teachers in a large number of cases and on reducing time spent on teaching mathematics in schools - echoing comments made in several annual reports on the state of mathematical sciences in Australia (see e.g AMSI, 2014; Attard, 2013).

The need to optimise time available in maths courses provides a second motive for using CAS. The crowding of school curricula has led to some reduction on teaching time for mathematics and, if this cannot be overcome, the care in using the time which we have is critical. In this regard decisions on what we teach and how we teach it become pivotal. Even in tertiary courses the amount of time that mathematics holds as a service course is under constant threat or erosion. The National Strategic Review (2006) commented on the following situation obtaining in Australia:

> "Less and less mathematics and statistics is being taught in many degrees and mathematical sciences departments continue to contract. Reduced mathematical content in many degrees has seriously weakened mathematical sciences in universities and is eroding Australia's skill base."

In Australia we have seen a serious decline in the numbers of mathematics staff employed in the major mathematics departments. In the major traditional universities this loss exceeded 30% over the decade from 1995 to 2005 [this was reported in the National Strategic Review of Mathematical Sciences Research published in December 2006] and this was reflected in the other universities. This report emphasises that mathematicians rightly focus on research in the field but we know most students in universities are not studying mathematics *per se* in elite departments but need it in service courses throughout the university system. Universities under pressure to optimise resources must make almost all decisions under financial constraints rather than educational grounds alone.

Mathematics has been drastically reduced in courses like MBA and business over time and faces erosion in engineering courses as it competes with the mainstream subjects. The overall time spent each year in tertiary courses has reduced with time also – we now have three fewer teaching weeks than we had 40 years ago adding to the impact on time available.

Risser (2011) noted some objections to CAS raised by the mathematics community involve balancing technology drawbacks and advantages. The list includes the time factor in teaching. Although not an opponent of the technology, Jardine (2001) argued that time spent teaching students how to use technology to solve mathematics problems is time *not* spent teaching students mathematics. Conversely, Buteau, Marshall, Jarvis and Lavicza (2010) argued the reduced time available is one motivation to increase use of CAS technology in tertiary courses. To meet the criticism of Jardine, the technology needs to be easily used and 'natural' as well as readily available and not especially syntax riddled. Available CAS platforms vary substantially in this issue of easy syntax.

Given mathematics coursework at university is under pressure and reform of content and teaching process could be usefully explored at least in context of service courses, it seems desirable to exploit as much technology as may assist in the development of concepts, the speed of delivery and the effectiveness of teaching. As usual the use in some assessment is desirable as it makes the incorporation of the tools more natural. This was the point being driven home by Leigh-Lancaster (2001) in his paper on use of CAS in final examinations for mathematics in Victorian secondary education. At present, restricting CAS access in university courses creates an unnecessary problem for students used to having them at senior secondary level which occurs in each state in Australia excepting New South Wales.

It is useful to compare the progress of technology use in mathematics courses with that in statistics courses where the advantage of technology use in statistics service subjects appears to be more widely recognised. In most universities, even in statistics subjects that do not use computer packages, calculators are commonly used to compute statistics such as Pearson's r, regression coefficients, the standard deviation, ANOVA, etc. Calculating regression coefficients or Pearson's product-moment correlation coefficient by hand is an extremely time consuming, tedious process and does not give students the required conceptual knowledge of how to actually apply and interpret these statistics, or the knowledge of when it is appropriate to use them. The entire process is a lengthy exercise in procedural understanding. This use of technology also provoked discussion. Moore (1997) argued that statistics and indeed mathematics would benefit from reform which embraced changes in pedagogy, content and technology use where all these aspects would reinforce one another. His argument drew on the changed character of tertiary students in an expanded system with most focussed on applications and data use. Although he noted that content and pedagogy should drive our instruction he observes that technology both serves these and changes what content may be appropriate.

A third motive for using technology including CAS lies in curriculum regeneration. The drivers of mathematics use in the modern world are the algorithms that enable suitable information to be distilled from the large amounts of data we can now collect. These are changing workplaces and even how we live with the internet and social networks embedded in lifestyle and dependent on such processes. The topics we can and should teach shift with these priorities with scheduling, simulation and operational research issues of more and broader impact now than the traditional issues of abstract algebra and continuous mathematics modelled so well by calculus. The depiction of the Simplex Method as the 'algorithm that runs the world' by Elwas (2012) headlined in New Scientist, underscores this perception. To teach all that we need to in undergraduate mathematics requires refocussing of content and more efficient ways of delivering the extant curriculum.

Moore (1997) makes this point in targeting reductions in formal probability courses as a way to allow expansion of other aspects of statistics. The role of simulation as an agent for teaching data based courses is substantial. A typical case would be formal aspects of queuing theory. Although some useful results can be found analytically by improbable assumptions (e.g uniform, exponential or Erlang distributions) on the patterns of distribution of service and arrivals, most queues really need a more informal simulation approach to get useful results. Simulation is a tool that assists in many areas of mathematical models from epidemiology to inventory, scheduling and other aspects of operations. Similarly our curriculum based on classical algebra and calculus needs to be efficient. For example, in service courses at least, once concepts are understood it seems unnecessary to examine every detail, such as how to anti differentiate every possible function which can be so treated.

3 ISSUES WITH TECHNOLOGY VARIANTS

If we decide to use CAS, the actual choice of CAS tool and how to include it also arises. We would argue that any CAS tool needs to be fit for the tasks required, easy to use and preferably cheap and portable for ready classroom use. In a recent survey pitting a Casio CAS calculator, (albeit an older model!) against three CAS computer tools, Hamilton (2012) found that the CAS calculator performed poorly on more than half of the test questions. The items of which were selected from the combination of the four test subject manuals. This reinforces the point that there is a serious difference in CAS options from high end dedicated products like Maple and Mathematica to the CAS calculators with restricted functionality. However this is also reflected in price differences and commonly in utility and even in the needs of the user as noted before. Each CAS tool has its place but they can hardly be expected to interchange! Also the time taken to master the CAS in each varies substantially and this affects use in class when they are being suggested as time savers!

In this paper we consider a variety of CAS tools which are common in Australian schools and universities. These include CAS calculators represented by the TI Nspire which is the most commonly used in Victoria and which gives similar results to the CASIO classpad which is also common. We also consider high end packages Maple and Mathematica which are often used in universities. We also consider Wolfram Alpha which is a cheap and convenient CAS package which is internet based – and certainly popular with our own students!

We illustrate this with a simple example of solving a basic first year undergraduate DE by a CAS calculator, by hand and by Wolfram Alpha, Maple and Mathematica.
We consider solving a second order inhomogeneous differential equation

$$y'' - 4y = e^{2x}$$

In Wolfram Alpha, the syntax is fairly natural. Explicit functional representation is useful here (i.e. $y(x)$ instead of y) but not always essential. The solution appears in a natural way, so long as the user is aware of exponential syntax etc.

In a modern CAS calculator, the TI-Nspire, we can enter this in fairly natural formulation. The only requirement is to know of the 'deSolve' feature at item D on the Calculus menu and the need for syntax where we signal the independent and dependent variable at the end, and in that order. Menu driven methods with use of primes in a natural way for derivatives gives easy syntax to navigate. This calculator returns the problem and solution only, unlike the more extended Wolfram Alpha output.

The actual output by CAS calculator is less transparent than that obtained by Wolfram Alpha. The student needs to understand the equivalence of the two results, namely that $c_1e^{2x} + c_2e^{-2x} + \frac{1}{4}e^{2x}x$ can be written as $\left(\frac{x}{4} + c_1 - \frac{1}{16}\right)e^{2x} + c_2e^{-2x}$ if we change the constants. The former is a far more natural format however and would be more sensible for the calculator to produce. If we solve by hand using an auxiliary equation and associated complementary function with a particular integral this is the style of solution we generate. Our experience has been that students take the output of technology seriously and are generally reluctant to change the format beyond simple relabelling of constants. In particular students will identify a 'particular solution' by simply removing the complementary function and so often will not give a minimal particular solution. Of course the result obtained is still a particular solution.

The problem was put to Maple and solved. Syntax is a serious constraint here but the solution is natural - allowing for writing constants after functions. For comparison we present the Mathematica solution of the same problem. Again far greater familiarity with syntax is required, appropriate for some student levels but unnecessary for most and requiring more teaching time! (See Figure 1.)

```
> ode := diff(y(x),x,x) - 4*y(x) = exp(2*x);
dsolve(ode);
```

$$ode := \left(\frac{d^2}{dx^2}y(x)\right) - 4\,y(x) = e^{(2x)}$$

$$y(x) = e^{(2x)}\,\_C2 + e^{(-2x)}\,\_C1 + \frac{1}{4}x\,e^{(2x)}$$

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[ >
```

Maple Syntax and Output

```
In[2]:= DSolve[{y''[x] - 4*y[x] == Exp[2*x]}, y[x], x]
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$$\text{Out[2]=}\ \left\{\left\{y[x] \to \frac{1}{16}e^{2x}(-1+4x) + e^{2x}C[1] + e^{-2x}C[2]\right\}\right\}$$

Mathematica Syntax and Output

Figure 1 Maple and Mathematica Output for the Differential equation

The commands, the capitals, the use of ==, even the unnatural square brackets could be a problem. It is interesting however that it yields a solution in the same 'unnatural' form as that from the Nspire TI CAS (and the CASIO Classpad). The process used by the CAS calculators and by Mathematica used another method from that demonstrated in Wolfram Alpha – most likely by Green's functions as Laplace transforms give the standard format.

CAS calculators are easy to master and menu driven whereas high end packages are expensive, syntax riddled and require extensive use to become easy to apply. This was noted in the work by Stewart, Thomas and Hannah (2005) in reporting student attitudes and instrumentation in the first two years of undergraduate maths courses. Hamilton also observes that a CAS calculator is 'personal and portable and has a very wide range of numerical calculation tools as well as symbolic' and noted that more modern CAS calculators may address the deficiency he found in his initial test model.

Use of CAS is not intended to exclude other technology. There are many technology tools available for use in courses and these also may have a role. Decker (2011) describes the role that java applets can play in demonstration of problems modelled by differential equations. He argues these are superior to CAS tools, either computer or calculator since they give rapid dynamic responses and multiple graphic outputs. For example changing the size of parameters in the logistic differential equation is readily explored by graphic means. Decker rightly notes where these applets improve over simple CAS solution of DE problems and in public domain such applets are a convenient addition to classroom teaching. However they are limited to the cases available and most students or mathematics staff would not be sufficiently proficient in java applet development to extend the range. Time taken to build that proficiency would detract from time spent on the material actually being studied and this would likely be a problem.

Mathematics serves a different role in each course. In a mathematics student's course it is central and every aspect of it should be explored. In such a case, teaching a course on differential equations for example requires inclusion of the subject theory including proofs on existence of solutions. In a service course role, mathematics is needed for functionality. The students in such courses are largely users of extant mathematics not creators of new mathematics processes even though they may be creative in application of existing methods. What we teach and the way we teach should be necessarily different. It is in these courses where the vast bulk of undergraduate mathematics occurs at university and in these courses where a more appropriate use of technology in the classroom and even the exam room should be considered. Paradoxically, Buteau et al (2010) found that CAS integration in tertiary mathematics classes occurs more frequently in mathematical majors rather than in service courses!

One fine example of using technology to assist higher mathematics is provided by Borwein (2009) in context of interaction with technology and other means to solve difficult problems. Borwein considers several examples where exploring with a computer can yield useful results mathematically. However, in his cases, this is at a level well above the usual service student!

It remains to consider what technology and how to use it effectively, what impact it can have and any evidence on current and past practice – both good and bad! One point needing emphasis is that repeated surveys over time show that students like using CAS calculators (Tobin and Weiss, 2011). The prospect of using technology to boost enjoyment of mathematics learning and motivation is one point made by Haapasalo (2013).

4 DISCUSSION

Following comment on repeated attacks on technology use and education practice by professional mathematicians in the US via the magazine Focus we examined the Australian Mathematics Gazette over recent years for similar 'official' lines. Most obvious are the many contributions which comment on the state of tertiary mathematics with technology mainly a minor player since it is largely banned in test assessment (see e.g. Crossley, 2006). Of course the reasonable view is one of balance. There needs to be sensible use of CAS or other technology not exclusive use. Students can certainly see the development of concepts separate from CAS usage even if they may later use it to shortcut work although it may also assist in conceptual learning.

We reviewed the Australian experience over the past 15 years in the professional literature and the press. The serious issues affecting mathematics in almost all the courses undertaken in senior secondary and tertiary undergraduate work provide some rationale for at least considering the expansion of technology use. The tipping point for most critics appears to be access in test assessment as use of packages like Mathematica, Maple and Matlab is commonly allowed in the tertiary sector provided its use is restricted to classroom or assignments. This naturally reinforces a retention of the classical curriculum and militates against a reconsideration of what should actually be taught and how much of it! It also requires teaching time be allocated to the high end packages and so removed from the actual coursework when time pressure is identified as a serious problem.

The remaining issue cited by Risser is fear of damage to student cognitive development induced by technology use. Arguments about procedural versus conceptual learning remind us that much of the time we teach technique and focus on procedure. Tall et al (2010) address this issue in their survey on Technology and Calculus. Noting that empirical research at undergraduate level is more limited than at school level, they still found research suggested use of technology to promote concepts while delaying procedural skills could lead to improvements in conceptual learning. In some studies no differences were found from classical learning groups but these appeared to be mainly those where technology use in experimental groups was not well integrated. This raises the issue again that simply adding technology to extant courses may give minimal advantage. Arslan (2010) points out how traditional instruction in differential equations courses emphasises procedural over conceptual learning – the reverse of what is generally preferred. Budinski and Takaci (2011) emphasise context in modelling where they introduced differential equations to final year secondary students by modelling contexts with the aid of the CAS features in their GeoGebra package. The aim is to boost conceptual learning overall with a possible sacrifice of extensive content, in the hope that learning transfer will improve. The role of technology in the balance of conceptual and procedural learning is taken up by Haapasalo (2013) with regard to the issue of how the technology affects the process. He suggests that adding technology in teaching and assessment alone will not likely change learning and puts a case for using the technology to assist in driving curriculum change – that other bogey raised by Risser.

Although critics of technology use in CAS calculators argue that there is much button pushing which can generate output with no comprehension or judgment of the result this can be similarly argued on many aspects of procedural learning undertaken by hand. Students can perform routine differentiation and integration with little feeling for the underlying concepts like rates of change. This can be even more so if extensive use of tables is involved. As noted, calculation of a correlation coefficient by hand for two data sets in statistics is no guarantee that the students would appreciate what this meant or whether it was even valid to perform. The time on calculation could be put to far better use. Similarly in calculus, for service course students it may be better to focus on procedure by hand on a reduced number of function types ensuring that concept development keeps pace and just using CAS for an extended set of functions. In this case we may save time enough to even expand the range of topics taught! This would do for Mathematics what Moore was urging for Statistics (Moore, 1997). Beaudin and Picard (2010) describe the experience of using CAS calculators in teaching mathematics courses over a decade and note that these can be used to give students more conceptual understanding although this is often not done if calculators are merely used to generate answers replicating traditional procedural learning. The balance between conceptual and procedural learning exercises educators but the assessment system commonly covers procedural learning much more.

Some use of handheld CAS technology can be popular and successful as Connors and Snook (2001) reported after its introduction to first year mathematics at West Point in the USA. As found at Swinburne University of Technology and Edith Cowan University in Australia that is no assurance of the process lasting! The best way to use this technology would be to write a curriculum around expectation that technology like it will always be available in future and that the focus on learning should shift to contexts, to applications, to learning of concepts that transfer well. It seems that we are far from being able to do this at present.

In this discussion we have considered not just technology use but the type used and the option to include this in some testing. We would argue it is not sufficient to restrict it to the classroom or even assignment use for reasons outlined although some conceptual testing with no technology is appropriate. We believe that high end CAS and even computer applets have their place but CAS calculators offer greater portability, economy, ease of use and avoid too much access to extraneous materials in a test situation. The different outputs CAS calculators can offer also lead to interesting discussions on alternative forms of solutions (e.g. in anti-differentiation) or different methods of solution (e.g. in differential equations). These surely promote thinking about the solution not merely obtaining it!

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BIOGRAPHICAL NOTES

Dr Patrick Tobin is a lecturer in Mathematics at Australian Catholic University. His current research interests include Operations Research and Computer Algebra Systems

Vida Weiss is a lecturer at Swinburne University and PhD student at Australian Catholic University. Her current research is focused on use of Computer Algebra Systems in mathematics education.